

Mathematical Knowledge for Teaching

Recent research in mathematics education has found a significant relationship between teachers' mathematical knowledge for teaching (MKT) and student achievement (Hill, Rowan & Ball, 2005). These researchers define MKT as the mathematical knowledge that teachers need to "carry out the *work of teaching mathematics*" (Hill et al., 2005, p. 373). Examples of the work of teaching mathematics include:

- Explaining terms and concepts to students
- Interpreting students' statements and solutions
- Using representations accurately in the classroom
- Providing examples of mathematical concepts, algorithms or proofs

This work requires "specialized" content knowledge, which Hill et al. (2005) argue is mathematical knowledge, not pedagogical knowledge. They suggest that mathematical knowledge is needed to assess the mathematical validity of various solution methods for a problem such as 45×65 or to determine how best to represent quantities such as $\frac{1}{3}$ or $.78$ using diagrams.

These research findings have direct implications for the preparation of elementary teachers. Elementary teachers need to develop their understanding of mathematical concepts and processes: they must be strong students of mathematics. In addition, elementary teachers must learn mathematics as a teacher, as they must be able to interpret the mathematical ideas in a student's comment and use multiple representations to unpack mathematical concepts. Thus, courses intended to develop teachers' mathematical knowledge need to reflect the distinctive demands that teaching places on teachers' content knowledge.

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