# Graduate Programs at UO: Requirements for entry and graduation

Elementary – Math 211-12-13 is a pre-requisite to graduate school. Within graduate school, there are 2 methods courses. [211-12-13 are all 4 credit courses… finally. It would be nice to have 5 credits per course to allow more use of manipulatives, computers, technology, etc. – CBMS says elem teachers "should be required to take at least 9 semester hours (14 qtr hours) on fundamental ideas of elementary school mathematics]

Middle – Basic Math Endorsement allows up to Algebra 1. For Graduate admission, students need to pass the basic test and requires 24 credits of college math (6 credits upper division). Inside Grad program, 2 methods courses as well as 2 additional courses (math wars and pedagogy). [CBMS says middle teachers "should be required to take at last 21 semester hours of mathematics (32 qtr hours), that includes at least 12 semester hours (18 qtr hrs) on fundamental ideas of school mathematics appropriate for middle grades teachers.]

High – Some can get in with basic endorsement (allowed to teach up to Algebra 1) so the middle school requirements are shown above. For advanced endorsement, requirement is passing Advanced math test and 36 credits of upper division math. Inside Grad program, 2 methods courses as well as 2 additional courses (math wars and pedagogy). [CBMS says undergraduate major or equivalent, with 6 hr capstone course connecting college math courses to high school mathematics].

[side note – students taking our "pure" or "applied" options would have 28 upper division math credits; students taking the "secondary ed" option would have 36 credits of upper division math, but at a lower overall difficulty level than the other two options.]

Methods courses are combined MS and HS, and instructors are expected to incorporate MS specific approaches within their courses. Math 211-12-13 is not required for MS or HS because it is not required for the Basic Math Endorsement. No specific courses are required for either Basic or Advanced (it would eliminate too many candidates). Most people doing the Basic are pairing it with another endorsement, often in the sciences.

Graduate program has option for electives. Math 605 is an upper division "reading" course, which I taught as an advanced methods course focusing on areas of weakness. We met 2 hours a day, 3 days a week, for 7 weeks. It was a 4 credit course and student comments are included at the end of this. The CBMS document *The Mathematical Education of Teachers* describes the ideal high school teacher as taking the "equivalent of an undergraduate major in mathematics, that includes a 6-hour capstone course connecting their college mathematics courses with high school mathematics."

The students spoke of feeling unprepared with multiple areas – most notably any area they were student teaching. Topics they wanted covered (in depth, with applications to high school teaching) were:

- Polynomials
- Rational Functions (including long and synthetic divison)
- Logarithms and Exponents
- Trigonometry – especially trigonometric identities and connections between them.
- Geometry (seemed a bit general to me)
- Probability (more) and Statistics (less)
- The History of Math
- Calculus
- Lesson plan ideas to tie things together
- Ways to integrate fun into the class
- Common themes/concepts

The first 2 weeks we covered some ideas across the curriculum, and had students bring in their proofs/ideas. $1 + 2 + 3 + \ldots + n$ was one. Showing that the central angle is 2 of the "slingshot" angle was another. Visual proofs were extremely fun, and we constantly looked for new ways to prove things.

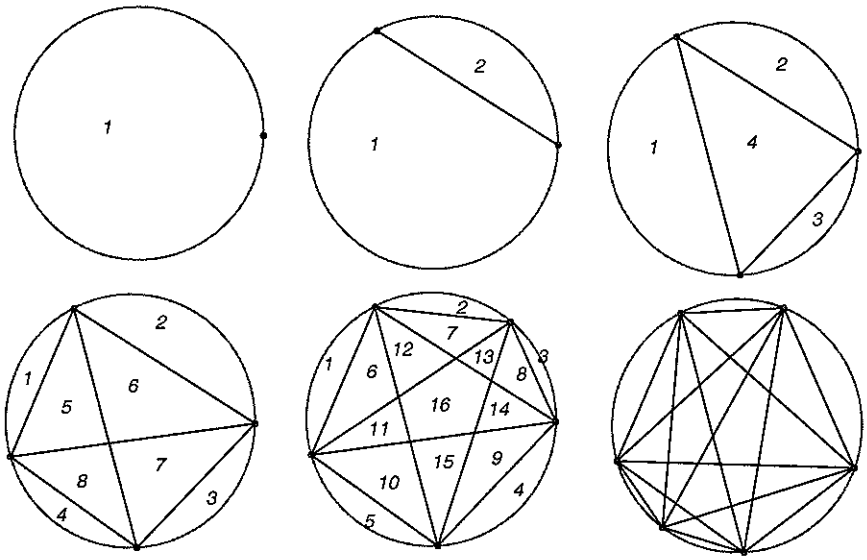
Later we linked our ideas to solving problems with spreadsheets. This was completely new to them, and their technology course didn't cover spreadsheets (or calculators). So we looked at problem solving, finance, sums of sequences, modular arithmetic, and Fibonacci numbers on Excel.

We took ideas like the arithmetic sequence and geometric sequences, and tied them to larger ideas. What about sums of these sequences? We generalized the sum of the arithmetic sequence $1, 2, 3, 4, \ldots, n$ in many ways, even geometrically, but didn't generalize the result (yet). Then we looked at the sum of the squares. (later we looked at the sum of the reciprocals of the squares and how that related back to $\pi$, and linked that problem to both calculus and probability). I asked them to find the sum of geometric sequences, then the sums of those.

Sums of sums provided interesting links. When you know the finite differences end 'nicely', like the $3^{rd}$ or $4^{th}$ finite difference became constant, then the degree of the polynomial was easy to find. Actually, we found that the polynomial was fairly easy to find as well.

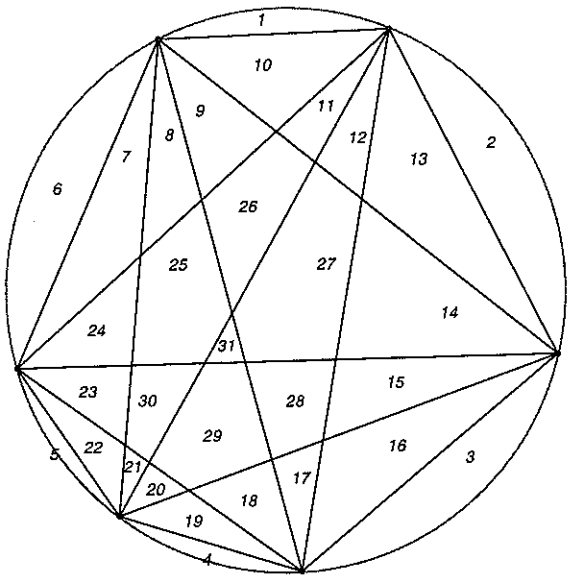
We extended to some bigger picture ideas. For example, we may have all seen this problem… but have you justified/proved it?

What is the maximum number of regions obtained by joining n points around a circle by straight lines?




So the pattern is 1, 2, 4, 8, 16, … I asked them what the next number was, and they responded, 32. Of course. So I asked them to justify it as part of their homework. Their justification was "because the pattern continues". Pressed further, one replied "it just has to."

So I assigned GSP and made them draw it and show their answer was either correct or incorrect. They eventually came up with a maximum of 31 regions – to their surprise and dismay. And I asked them to justify/prove their answer. This process took almost 3 weeks for them to really understand and be able to explain the idea… but it was time well spent.



A000127 Maximal number of regions obtained by joining n points around a circle by straight lines. Also number of regions in 4-space formed by n-1 hyperplanes. (Formerly M1119 N0427) +20 43

**1, 2, 4, 8, 16, 31**, 57, 99, 163, 256, 386, 562, 794, 1093, 1471, 1941, 2517, 3214, 4048, 5036, 6196, 7547, 9109, 10903, 12951, 15276, 17902, 20854, 24158, 27841, 31931, 36457,

A133552 Number of length n binary sequences with at most 4 of every adjacent 6 bits set. +20 4

**1, 2, 4, 8, 16, 31**, 57, 109, 209, 401, 769, 1473, 2817, 5391, 10321, 19761, 37834,

A118891 Number of binary sequences of length n with no subsequence 01110. +20 3

**1, 2, 4, 8, 16, 31**, 60, 116, 223, 428, 820, 1569, 3002, 5744, 10992, 21039, 40273,

A000128 A nonlinear binomial sum. (Formerly M1120 N0428) +20 1

**1, 2, 4, 8, 16, 31**, 58, 105, 185, 319, 541, 906, 1503, 2476, 4058, 6626, 10790, 17537,

A006775 Number of n-step spirals on hexagonal lattice. (Formerly M1121) +20 1

**1, 2, 4, 8, 16, 31**, 61, 115, 213, 388, 691, 1218, 2110, 3617, 6113, 10238, 16945, 27802,

A018763 Divisors of 992. +20 1

**1, 2, 4, 8, 16, 31**, 32, 62, 124, 248, 496, 992

A104993 Molien series for a certain 16-dimensional group of order 20160. +20 1

**1, 2, 4, 8, 16, 31**, 61, 117, 224, 424, 796, 1476, 2717, 4938, 8876, 15756, 27616, 47764,

Using the triangular numbers to find a general sequence was one that most of them really enjoyed (they called it the 'cool' method). I based some of this off Al Couco's book and his ideas about finite differences. I required them to purchase an older (cheaper) edition of Posamentier's Methods for Teaching High School Mathematics book which was new to them – they didn't have a methods book.

Or the Fibonacci numbers… which led to many new ideas for them. I asked them to prove the derivation of Binet's Formula – which turned out to be significantly easier than all of them made it. When they couldn't do it immediately, it was part of their homework. When the homework failed, I had them google it. When they couldn't discern google, then we talked about it and I guided the learning. They would teach each other and often found different topics interesting.

We did look further at many topics, but the sums idea was a constant one. We also looked into linking $\pi$ to non-geometric areas, including integrals with semi-circles on a right triangle. It was a good review of calculus which they admitted to being rusty on.

Pedagogically, we expanded geometric areas into pieces that were easier to see. Their comments and questions were extremely good and to the point – this would be after their student teaching so they already had a few successes and many failures. Their term ended with them doing some research into journals – finding topics of interest to them. They had to write a paper summarizing the course (more for me than them) as well as join OCTM to get journals sent to them.

- Asking "what if" was a constant.
- Having them be actively engaged by suggesting topics was extremely helpful.
- Give daily homework, with extensions to other areas (in more depth) for those interested.
- Asking students to "find something"
- Bringing technology into play was critical for some students who didn't have the 'intuition' or 'experience' of others.
- Dealing with their colleagues was more challenging than they thought. Many were negative about any other methods than their 'best' method. Only one had a master teacher who didn't use traditional methods of teacher speaks – students do. There was a lot of frustration about this being new teachers and that their methods were looked down on.
- Making them present topics to each other held them accountable.

The last piece of the term was a list of good (fun) non textbooks that were math based. They were eager to learn anything, especially history, and wanted to know about good journals, website resources, etc. I compiled what I could into a document and sent it to them after the grades were entered.

## Student #1

Trials and Tribulations of the Second Summer Session

I am not going to start this paper by telling you what I subjects I liked or disliked. First, I'm going to explain why this course was so helpful to me. Before I started student teaching, we had discussions in our methods classes as well as the independent study classes with Shlomo about the importance of showing or leading the students to discover why math works. When I got into the schools and started teaching, I did not see one teacher showing the students why the things they were learning about in math worked and it was more than a little discouraging. After the first two weeks, I gave up on trying to teach that way, because my cooperating teacher didn't give me any support and discouraged me from teaching like that. I think that this course really helped me to realize that it can be different for me when I am practicing as a teacher. I didn't know if it were really possible or if it was just something all of the methods teachers and Shlomo had to tell us. The course really made me realize how much more interesting all of the things we were talking about would be to the students, rather than simply having them copy down notes and practice a few problems. Again, I had heard the arguments before about why that is bad from various other educators. It really hit home in this class because you were very passionate about it and every topic was introduced in the manner of which I would want to present it to my students. I just think you took the time to hold meaningful conversations about why it is so important that you present things in math in a very intentional and specific way. I really appreciated those discussions.

The topics I really enjoyed were the problems that we solved using an excel worksheet. I had never worked with excel like that before and it was a really powerful tool for solving problems. I also really enjoyed the various proofs of the law of cosines and the proofs of the Pythagorean theory. It was good to hear you emphasize that you should avoid saying if the angle is 90 degrees, then we have the Pythagorean theorem, because I pointed that out to my own students but see the importance of this simply being a particular case.

The only topics that didn't interest me were the problems we were doing with the differences of geometric and arithmetic sequences. It wasn't that the math wasn't interesting, I just didn't ever have to teach about sequences and was unsure of when they are presented in school. Therefore, it was hard for me to think about how I would teach about these problems. If I had some more context as to what class they are presented to students in and a textbook with how the material is sequenced then I would have had a better understanding of how I would use these problems in my own classes.

## Student #2

Becoming more prepared to teach high school students was a corner stone of the math reading class. I enjoyed the time you put into acquiring resources that you believed to be helpful to you as well. Knowing what organizations to join, as well as what magazines to subscribe to, helped when trying to come up with ideas to motivate our students. I personally enjoyed all the work we did with the Golden Ratio and the Fibonacci sequence. Working problems and finding other areas that these numbers come up was intriguing and you showed us problems that I had not seen prior. The technology aspect was beneficial as well, seeing the ways something as basic as Excel could be used in the classroom gave a lot of ideas that I will try to incorporate as well. Finding the difference of differences and the ties to calculus i.e. given four point find an equation that fits the data, is an approach to tying different ideas together that I will use in my advanced

classrooms. The geometry problems you posed as well, learning to ask what if this is changed how does the problem change. Learning to ask more questions once you solve a given problem I thought was tremendously valuable. The most valuable to me was allowing us to decide what areas we felt we needed to cover. We all chose areas that we were either going to teach in the future or we felt the weakest in. In future classes these areas may change as we all are different but allowing us that freedom was the most beneficial.

## Student #3

Summer '10- Math 605

As I was looking back through my notes and homework problems from this summer, I saw a lot of very interesting problems and questions that we discussed. For example I really liked looking at sequences and finding an arithmetic sequence after the $2^{nd}$, $3^{rd}$ etc. difference, and being able to find an equation using our "cool" method. I also liked finding the sum of a geometric sequence using a similar method as finding the sum of an arithmetic sequence. I liked all the sequence stuff that we did because it took many different topics and yet we kept finding the triangular numbers show up. As a student in this class, I really enjoyed seeing the connections between topics, but as a teacher, I would have liked to discuss more how to integrate this strategy into the classroom.

I have no complaints with anything that we covered this summer; however I would have liked to look at derivatives and where they come from and real world applications of them. I also would have liked to look at more math history. I want to build a bank of stories about famous mathematicians that makes the study of math more interesting.

I think the best thing we did this summer was doing work with excel because I had not used it before and there are a lot of applications where it can be used, in particular the financial applications. It is very practical to look at my student loan debt and see how much money I can save by paying a little more each month.

The last thing I want to say about this summer class was that I really the ways that you tried to find ways to make the material interesting and engaging for our future students. I liked the problems that were not too difficult for us to solve because it allowed us to look at different ways of presenting those topics and looking at ways that might make more sense to students. Similarly, I liked the math "tricks" as ways to get students curious about math and then using that to explore some very basic topics like counting and combining numbers.

I hope this type of class will be an option for future cohorts because this was a much more valuable class than any of the other options I had for an elective this summer. Thank you for all the work you put into this class this summer and providing us with a list of books and resources. If this class continues for future years, I do not think the specific topics that we covered are essential for the success of this class instead I would focus on showing the relationships between topics that the future students are interested in.

I hope my feedback was beneficial. I really enjoyed the class this summer, and I like that you brought in topics that we didn't specifically mention at the start of the term because I know there are plenty of topics that I do not know about to know that I want to know more about them, if that makes sense.

[This student also emailed me the following: Thanks for all the work you put into this class this summer. If you are going to talk to people in the Ed. department, you should try to get the math 211-213 sequence required for

secondary math teachers too because I think that was the first time I started really thinking about how to explain some of those topics.]

## Student #4

This class was an excellent way to wrap up everything we have learned in the UOTeach program. We discussed how to implement problem-solving and problem-posing in our classrooms, as well as how to explore and develop advanced topics. Two of the keys to the success of this class were our ability to work with just a few others who shared similar interests, and the fact that it came at the end of the program.

I really enjoyed our methods courses during the Fall and Winter terms and I learned a lot from them, and this class was able to go the next step. Our methods courses blended individuals from all different interests and mathematical background, as well as basic and advanced math teachers. This was definitely an asset to these classes, and I would not change it, but I really liked how this Math 605 course was able to focus in on the common interests within our small group. Often during the general math methods classes, much of the focus is understandably on the basic math, while those of us interested in advanced math are left to figure out the connections for ourselves. In this class, we got to see interesting problems related to Geometry and beyond, and we got to study these topics in more detail. Not only did we study them for ourselves, but we also discussed how to relate the material to our students.

I especially enjoyed our discussions on trigonometry, and how to develop this tricky topic for our students. Now I feel prepared to provide my students with carefully scaffolded lessons that lead to the discovery of these functions, which is something we talked about a lot in UOTeach. I also feel that the many cross-subject connections that we made were extremely helpful, such as connecting polynomials to series and series to spreadsheets.

This class was very well structured in terms of how the topics were introduced. For example, before we explored the problem of cutting a circle into regions, we talked about how to find the sums of squares and cubes. This was an integral tool in solving the circle problem rigorously, which is an important concept to keep in mind in a classroom. We got to see how seemingly unrelated topics correlate and evolve into one another; nothing in the class was totally disjoint from everything else.

This class was essentially another methods course, albeit one that went beyond what our other ones did, but a big part of the reason it was so effective was that it came after our other courses and student teaching. We were able to bring real problems and real successes that we had had in our classes into this course, and this drove much of what we studied. Also, we already had been exposed to the ideas in our previous courses, so now we were able to take them further. We started off last summer discussing the ideals of problem-solving and inquiry-based learning, then we got to try some of it out during the school year, and now we were able to come back and apply what we had learned as teachers to that ideal we strived for last summer.

This course also helped reinforce the idea in my mind that we should explore in great detail those problems that interest us, and pass this enthusiasm along to our students. We had homework problems that everyone was assigned to do, but many had another level that we could explore if it piqued our interest. This made it so that we all were on the same page, but we were not forced to do an excessive amount of work. This is a model that I would really like to try and employ in my own classroom someday.

Taking this class was a great decision. It really solidified everything we had learned in the program thus far, and was very well taught. The only downside I see is that if more people

want to take it in the future, we risk losing the benefits of it being a small group. However, I believe that it would be well worth any future math teacher's time to take this course.

## Student #5

Beyond anything we discussed in the class, I appreciate you suggesting the Posamentier book. It is a resource I will likely continue to use, even after I've gained several years of experience. In addition to the many valuable ideas for interesting lessons in the second half, the first half of the book nicely covers nearly all of the issues that I wish the methods classes would have discussed (structuring cooperative learning and group work, motivation techniques, effective questioning, problem solving approaches, using technology, extracurricular activities, etc...).

My favorite section of the course was the time we spent using Excel to explore some financial mathematics and sequences. It was interesting and fun to work with for me, and I think my students will really enjoy using spreadsheets to explore mathematical concepts. There is even a section in the Posamentier book with ideas for using spreadsheets in math class.

"Algebra for convenience" was one of the more interesting and valuable ideas you introduced. I think anytime we show students more efficient or interesting ways to look at/solve ordinary problems, it peaks their interest and opens them up to the mathematical concepts behind the methods. The same can be said for the "cool method" we used to find a general equation of a function, given some of the input and output data. I liked your ideas for "discovering" pi, polynomial dancing, and having students explore order of operations by coming up with their own rules.

If I were to take this course from you again, I would like to hear more about the strategies you use in your classes with prospective elementary school teachers. We focused mostly on lessons and activities for students who have higher levels of interest in, and proficiency with math. I would like to hear some of the ways you work with the students who can't stand math. Do you have students who come into your class hating math and leave with a different outlook?