2013 TOTOM Conference

Exploring “off the peg” geoboard shapes

Matt Ciancetta
Western Oregon University

Geoboards

- Geoboards and Geobands have already been distributed.
- I believe that some of you are already “playing” with your manipulative. Yes?
- Would anyone like to share interesting shapes or patterns they made?

Great! Thanks!

Geoboard’s variety of uses

- As a brief review/introduction, let’s brainstorm all the ways you have used geoboards.
- I’ll get it started…
  - Points, line segments, angles,
  - polygons, # of sides & angles of a polygon,
  - …

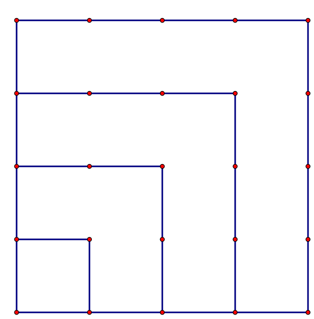
Explorations with sets of polygons

- Creating sets of polygons to use for exploring certain topics or concepts is one of my favorite styles of geoboard activities.
- How about activities that launch by creating all the squares on a 5-pin x 5-pin geoboard?
  - What are some possibilities for where this exploration may lead?

Exploring sets of squares

- When constructing the set of all non-congruent geoboard squares,
  - there is typically a set of squares that is obvious and students usually find with no problem.

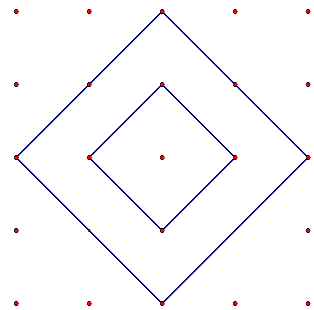
Exploring sets of squares



Exploring sets of squares

- When constructing the set of all non-congruent geoboard squares,
  - there is typically a set of squares that is obvious and students usually find with no problem,
  - then there is a set of squares that are less obvious but, still, many students find with enough time to think and explore

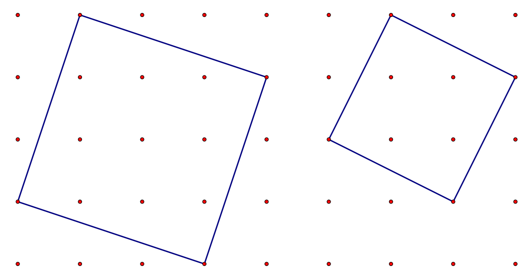
Exploring sets of squares



Exploring sets of squares

- When constructing the set of all non-congruent geoboard squares,
  - there is typically a set of squares that is obvious and students usually find with no problem,
  - then there is a set of squares that are less obvious but, still, many students find with enough time to think and explore,
  - and finally there is a set of squares that is not obvious to most students.

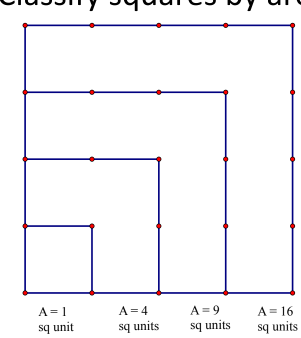
Exploring sets of squares



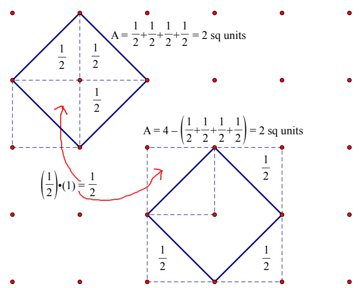
Exploring sets of squares

- How do you know that all your squares are non-congruent? Can you show or prove it?
- Classify the squares by area.
  - Volunteers?

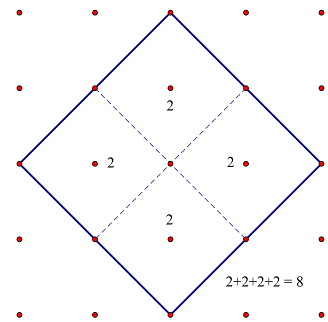
Classify squares by area



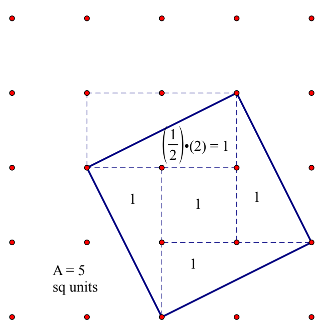
Classify squares by area



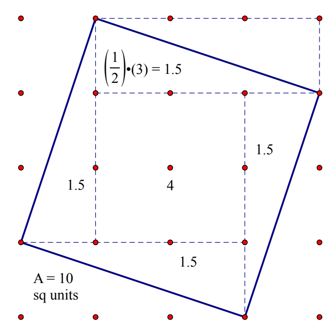
Classify squares by area



Classify squares by area



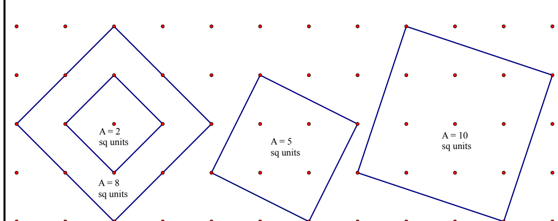
Classify squares by area



Classify squares by area

- So we found squares of areas:
  1, 2, 4, 5, 8, 9, 10, 16

Wait a second…do we really know/believe that all these are actually squares? Even those “tilted” squares?



Finding Squares extension

- Let's revisit the general geoboard "rules" …

OK, the main rule is that line segments are formed by stretching geobands around pegs.

So, for polygons, the vertices can only be located on the pegs.

This is important especially when students transition to sketching geoboard polygons on paper – you cannot just plunk down a vertex between pegs!

Finding Squares extension

- Hold everything!
- What do we know about two lines that intersect?
- Let's look back at some of your shapes created from playing around (at the beginning of the talk). Did you identify any points are off the pegs? Could those points be vertices of a polygon?

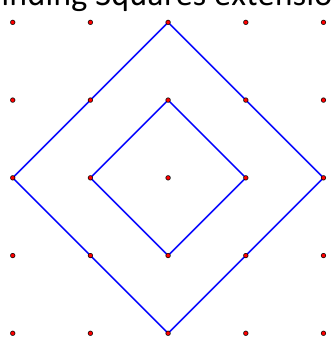
Finding Squares extension

- Use your physical geoboards to create *off the peg* geoboard squares.
- Can you find squares with
  - only 1 vertex off a peg?
  - 2 vertices off pegs?
  - 3 vertices off pegs?
  - 4 vertices off pegs?
- Share your results … are they really squares?

Finding Squares extension

- There are multiple non-congruent squares whose sides are oriented similarly, i.e., with the same two slopes.
- For example. Let's look as some of our *on the pegs* squares. We previously constructed two non-congruent squares that utilize slopes of +1 & –1.

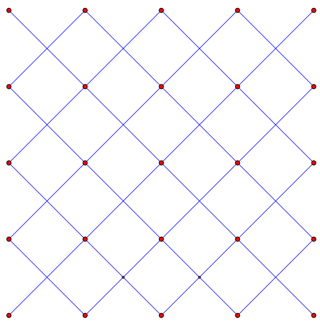
Finding Squares extension



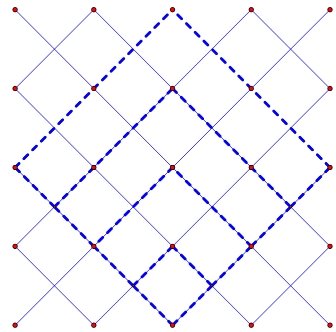
Finding Squares extension

- Are there more *off the pegs* squares that have sides with slopes +1 & –1?
- Can you make a construction to show all of these squares?

Finding Squares extension



Finding Squares extension



Finding Squares extension

- Review what you have and then let's determine all the possible "slope families of squares".

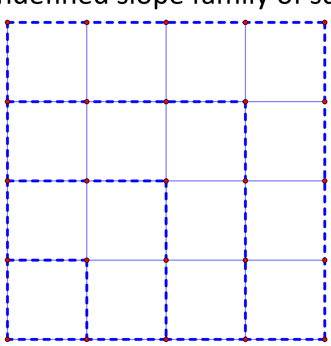
The families are:

- 0 & undefined ✓
- 1 & –1 ✓
- 2 & – $^{1}/_{2}$ ✓
- 3 & – $^{1}/_{3}$ ✓
- 4 & – $^{1}/_{4}$ ✓
- $^{3}/_{2}$ & –$^{2}/_{3}$ ✓
- $^{4}/_{3}$ & –$^{3}/_{4}$ ✓

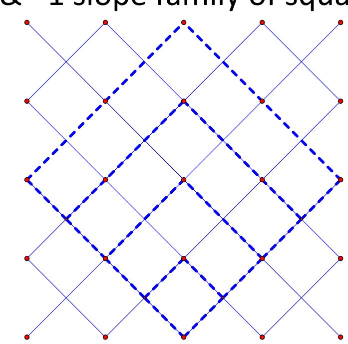
Finding Squares extension

- For each slope family can we sketch a single a construction to show all of the squares in each family?
- We already have constructions for the 0 & undefined and 1 & –1 families.

O & undefined slope family of squares



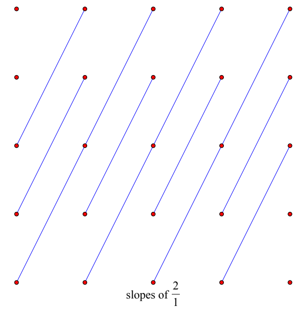
1 & –1 slope family of squares



slope families of squares

- Constructions for the other families?
- Volunteers?

2 & – $^1/_2$ slope family of squares



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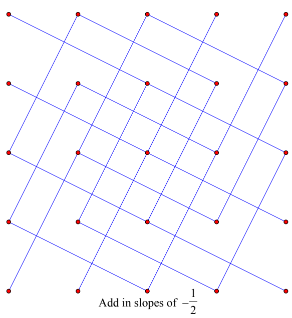

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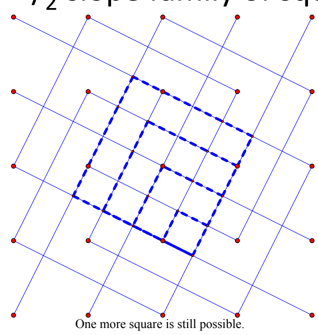

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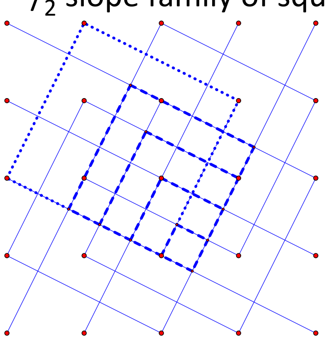
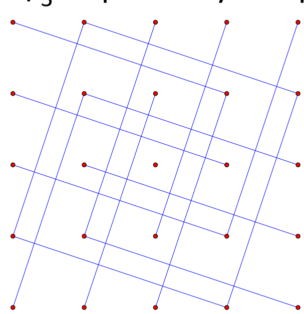
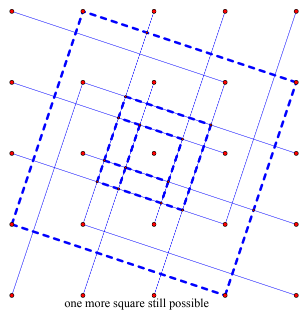


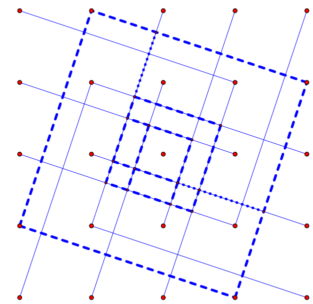
3 & – $^1/_3$ slope family of squares



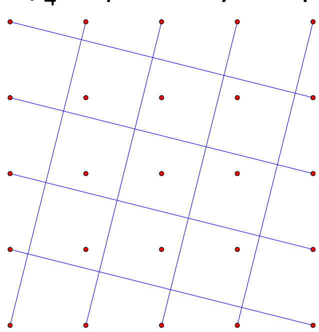
3 & $-{}^{1}/_{3}$ slope family of squares



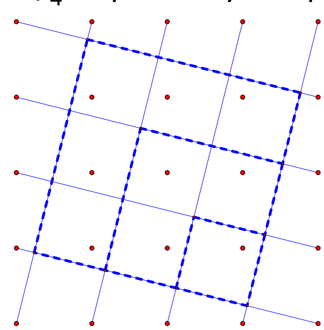
3 & $-{}^{1}/_{3}$ slope family of squares



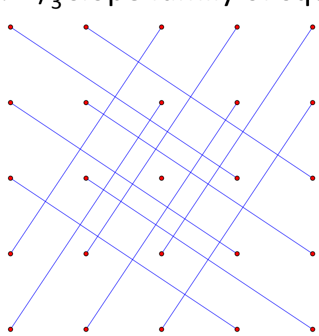
4 & $-{}^{1}/_{4}$ slope family of squares



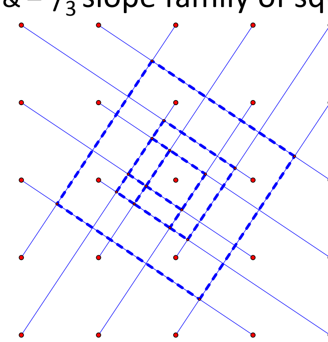
4 & $-{}^{1}/_{4}$ slope family of squares



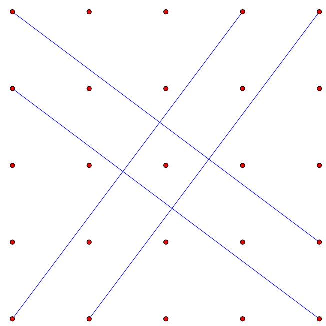
${}^{3}/_{2}$ & $-{}^{2}/_{3}$ slope family of squares



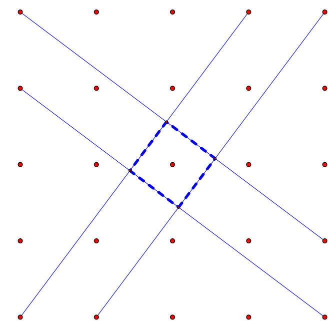
${}^{3}/_{2}$ & $-{}^{2}/_{3}$ slope family of squares



$^4/_3$ & $-^3/_4$ slope family of squares



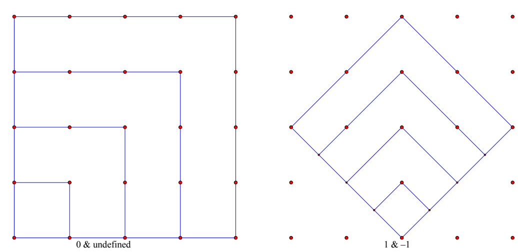
$^4/_3$ & $-^3/_4$ slope family of squares



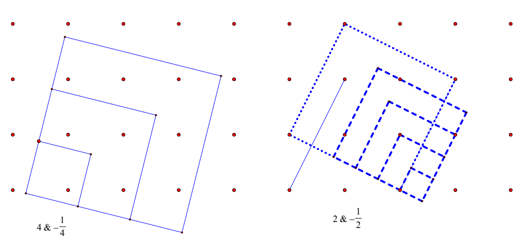
Finding Squares extension

- Now that we have what we hope are complete families of squares, we may ask ourselves,
  - How many non-congruent squares in all are possible?
  - Are all the squares really non-congruent? What are the areas of those squares?
- Look for and share some patterns you see in each family. Share.
- Try to determine areas without converting to coordinate geometry.

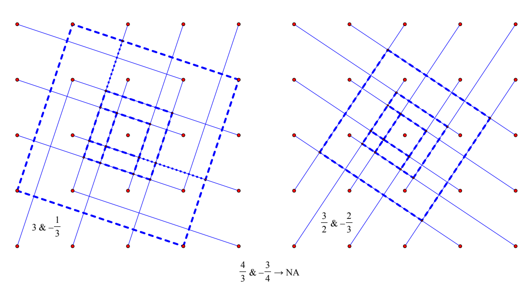
Area Patterns



Area Patterns



Area Patterns



Area Patterns

- Areas within each family increase by multiplying by perfect squares!
- 0 & Undef; 1 & –1: area of smallest multiplied by 1, 4, 9, 16
- 2 & $-{}^{1}/_{2}$: area of smallest multiplied by
  1, 4, 9, 16, 25
- 4 & $-{}^{1}/_{4}$: area of smallest multiplied by
  1, 4, 9

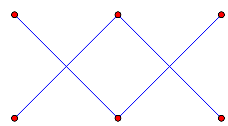
Area Patterns

- 3 & $-{}^{1}/_{3}$: area of smallest multiplied by
  1, 4, 9, 16, 49, 100
- ${}^{3}/_{2}$ & $-{}^{2}/_{3}$: area of smallest multiplied by
  1, 4, 9, 16, 64
- ${}^{4}/_{3}$ & $-{}^{3}/_{4}$: one one square (area mult by 1)

Areas of squares

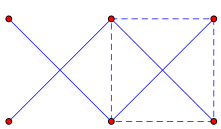
- Great! So now all we need to do is find the area of the smallest square in each family and multiply it appropriately to determine all the areas.
- 0 & undefined family:
  - Areas: 1, 4, 9, 16
- Let's look at the 1 & –1 family

1 & –1 family



What is the area of the smallest 1 & –1 square?

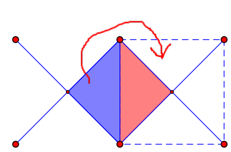
1 & –1 family



What is the area of the smallest 1 & –1 square?

Compare it to a square with known area, say a square of 1 sq unit

1 & –1 family

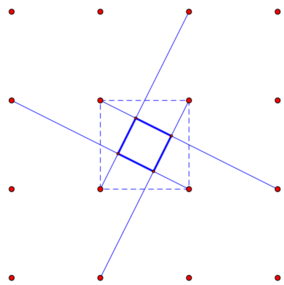


What is the area of the smallest square?

It takes 2 small squares to fill in 1 unit square.

1 small sq = ½ sq units.

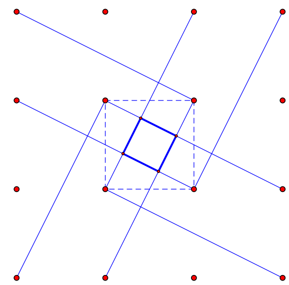
2 & $-{}^{1}/_{2}$ family



What is the area of the smallest square?

Compare it to a square of 1 sq unit

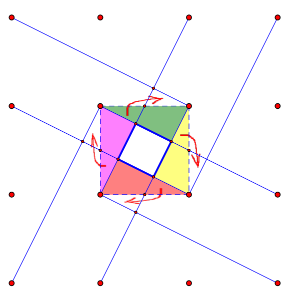
2 & $-{}^{1}/_{2}$ family



What is the area of the smallest square?

How many small squares are needed to “fill in” a unit square?

2 & $-{}^{1}/_{2}$ family

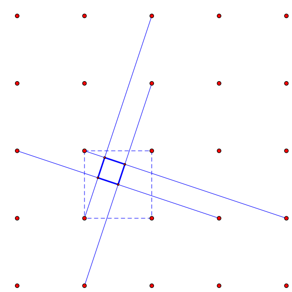


What is the area of the smallest square?

It takes 5 small squares to fill in 1 unit square.

1 small sq = ${}^{1}/_{5}$ sq units

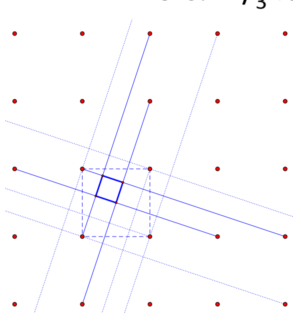
3 & $-{}^{1}/_{3}$ family



What is the area of the smallest square?

Compare it to a square of 1 sq unit

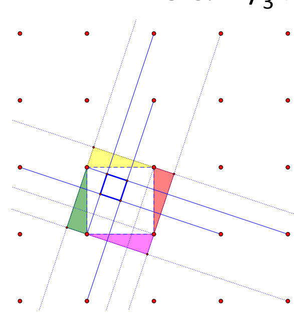
3 & $-{}^{1}/_{3}$ family



What is the area of the smallest square?

How many small squares are needed to “fill in” a unit square?

3 & $-{}^{1}/_{3}$ family



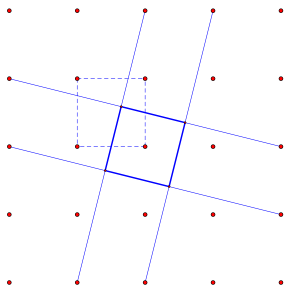
What is the area of the smallest square?

16 – (1.5+1.5+1.5+1.5) = 10

So, it takes 10 small squares to fill in 1 unit square.

1 small sq = ${}^{1}/_{10}$ sq units

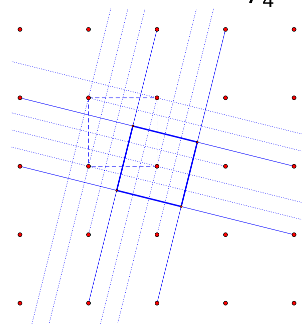
4 & $-{}^{1}/_{4}$ family



What is the area of the smallest square?

Compare it to a square of 1 sq unit

4 & $-{}^{1}/_{4}$ family

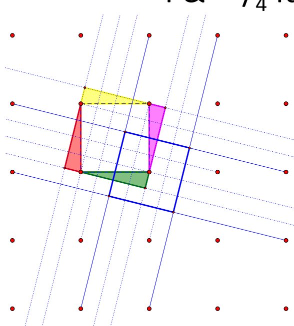


What is the area of the smallest square?

Cut this smallest square into 16^{ths}.

How many of these 16^{th} pieces cover the unit square?

4 & $-{}^{1}/_{4}$ family

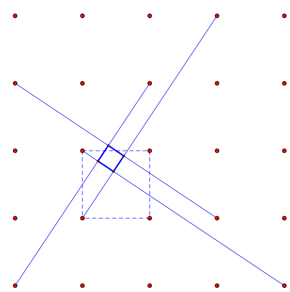


What is the area of the smallest square?

25 – (2+2+2+2) = 17
So, 17 of the 16^{th} pieces cover the unit square.

1 small sq = ${}^{16}/_{17}$ sq units

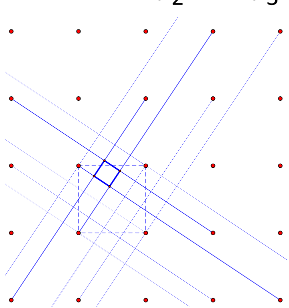
${}^{3}/_{2}$ & $-{}^{2}/_{3}$ family



What is the area of the smallest square?

Compare it to a square of 1 sq unit

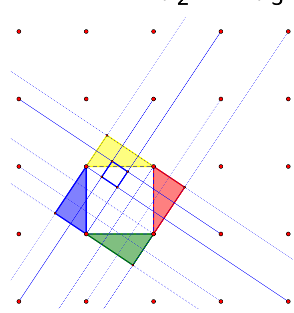
${}^{3}/_{2}$ & $-{}^{2}/_{3}$ family



What is the area of the smallest square?

How many small squares are needed to “fill in” a unit square?

${}^{3}/_{2}$ & $-{}^{2}/_{3}$ family

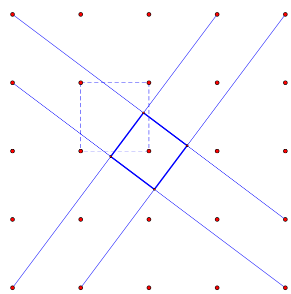


What is the area of the smallest square?

25 – (3+3+3+3) = 13
So, it takes 13 small squares to fill in 1 unit square.

1 small sq = ${}^{1}/_{13}$ sq units

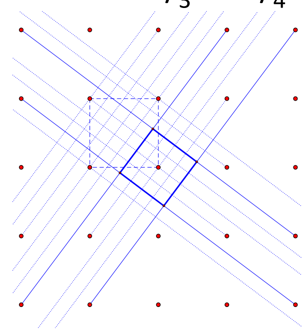
$^{4}/_{3}$ & $-^{3}/_{4}$ family



What is the area of the only square?

Compare it to a square of 1 sq unit

$^{4}/_{3}$ & $-^{3}/_{4}$ family

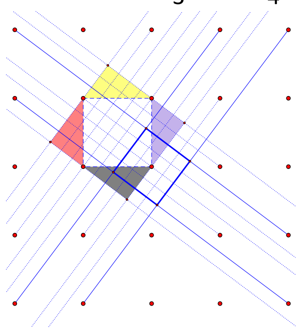


What is the area of the only square?

Cut this only square into 16ths.

How many of these 16th pieces cover the unit square?

$^{4}/_{3}$ & $-^{3}/_{4}$ family



What is the area of the only square?

49 – (6+6+6+6) = 25
So, 15 of the 16th pieces cover the unit square.

1 small sq = $^{16}/_{25}$ sq units

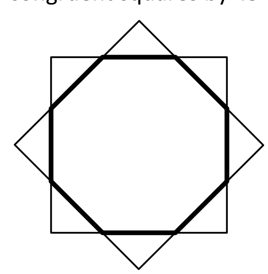
Areas of squares

- 0 & Undef; **1**, 4(1) = **4**, 9(1) = **9**, 16(1) = **16**
- 1 & –1: $\mathbf{^{1}/_{2}}$, $4(^{1}/_{2}) = \mathbf{2}$, $9(^{1}/_{5}) = \mathbf{^{9}/_{2}}$, $16(^{1}/_{2}) = \mathbf{8}$
- 2 & $-^{1}/_{2}$: $\mathbf{^{1}/_{5}}$, $4(^{1}/_{5}) = \mathbf{^{4}/_{5}}$, $9(^{1}/_{5}) = \mathbf{^{9}/_{5}}$, $16(^{1}/_{5}) = \mathbf{^{16}/_{5}}$, $25(^{1}/_{5}) = \mathbf{5}$
- 3 & $-^{1}/_{3}$:
$\mathbf{^{1}/_{10}}$, $4(^{1}/_{10}) = \mathbf{^{4}/_{10}}$, $9(^{1}/_{10}) = \mathbf{^{9}/_{10}}$, $16(^{1}/_{10}) = \mathbf{^{16}/_{10}}$, $49(^{1}/_{10}) = \mathbf{^{49}/_{10}}$, $100(^{1}/_{10}) = \mathbf{10}$
- 4 & $-^{1}/_{4}$: $\mathbf{^{16}/_{17}}$, $4(^{16}/_{17}) = \mathbf{^{64}/_{17}}$, $9(^{16}/_{17}) = \mathbf{^{144}/_{17}}$
- $^{3}/_{2}$ & $-^{2}/_{3}$:
$\mathbf{^{1}/_{13}}$, $4(^{1}/_{13}) = \mathbf{^{4}/_{13}}$, $9(^{1}/_{13}) = \mathbf{^{9}/_{13}}$, $16(^{1}/_{13}) = \mathbf{^{16}/_{13}}$, $64(^{1}/_{13}) = \mathbf{^{64}/_{13}}$
- $^{4}/_{3}$ & $-^{3}/_{4}$: $\mathbf{^{16}/_{25}}$

- We constructed 28 non-congruent squares on a 5-pin x 5-pin geoboard.
- Now the world is our oyster!
- This exploration could continue on in a variety of directions.
- Investigating octagons might be fun.
- This presentation is based on the collaborations of Steve Blair, Dan Canada, & Matt Ciancetta

Geoboard Octagons

- One way to construct a regular octagon is to offset two congruent squares by 45˚



Geoboard Octagons

- One way to construct a regular octagon is to offset two congruent squares by 45˚
- Doesn't it seem like we should be able to do this with at least one of the 28 squares we just constructed?
- Unfortunately we cannot. But, we can construct octagons with congruent sides.
- Then we could construct sets of similar octagons, and then…

Geoboard Octagons