Coherence and Mathematical Practices in Preservice Preparation

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Purpose

The Common Core prescribes significant shifts in the teaching and learning of mathematics. How are those being reflected in our preservice preparation?

We will take two aspects of the Standards, namely engagement in the Mathematical Practices and Coherence, and share how we address them in our 21X sequence at UO.

Content focus: work in other bases

Using different bases to represent numbers has often been included in preservice elementary content courses. Traditionally the rationale has been to help ensure that the students understand the standard algorithms of arithmetic.

Some reformers see this as of less value than other activities, and have jettisoned the topic.

We have been developing a more coherent use of different bases, in part with an eye towards deeper reflection on the Common Core.

First exercise

Skip count by fours in base eight – while doing jumping jacks!

Then skip count by threes in base eight (still while doing jumping jacks).

Shifts in our teaching

- When working in other bases, use unit-form names.
- When working in bases greater than ten, use circle (or O) notation.
- Highlight activities such as skip counting, counting on, multiplying by the base, and using area models, which are prescribed by the Common Core.
- "Fill in the number line" in the order the Common Core prescribes.
- Have students reflect on the mathematical work being done, which they can "feel" themselves when they work in other bases.

Second exercise

- Take the number 23 and write it, then double it, then double that, etc. a total of six times.
- Take the first number on your list (namely 23), the fourth number and the sixth number, and add them together.
- Compare what you got in the previous step with 23×41.
- Compare the fourth number on the list to 23×8, and the sixth number to 23×32.
- Explain what is going on in the previous calculations.
- Using your list, can you quickly calculate 23×18?
- Extend your list if necessary to calculate 23×75.
- Explain how to compute 23 multiplied by any number, or in fact do any multiplication, by "doubling and addition."

Mathematical Practices

For adult learners (and depending on implementation), which Mathematical Practices are likely to be invited by this activity?

Wait a minute...

Where are the other bases?

Work through a solution... (and note that students are provided even more scaffolding than shown here.)

Digression on Meaning-Method-Mastery

This activity comes after the introduction of other bases, and their use in addition.

By brining in base two in a situation which doesn't at first call for it, we are reinforcing **mastery** through application, in this case to further mathematics.

Digression on Meaning-Method-Mastery

In one model, the three stages of a learning flow are:

Meaning – Understanding the meaning of a concept

Method – Developing machinery (skills) that allow us to work fluently with the concept

Mastery – Demonstrating fluency, and using the concept to do more advanced mathematical or real-world problems

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The Common Core almost never prescribes skills without first developing a conceptual foundation to support them, or without having a future purpose for them.

Example: 3.MD Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

Meaning 3.MD.5. Recognize area as an attribute of plane figures and understand concepts of area measurement.
a. A square with side length 1 unit, called a unit square, is said to have one square unit of area, and can be used to measure area.
b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.

Example: 3.MD Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

Method

3.MD.6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
3.MD.7. Relate area to the operations of multiplication and addition.
a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

Digression on Meaning-Method-Mastery

Example: 3.MD Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

Mastery 3.MD.7 b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a+ b and a+ c. Use area models to represent the distributive property in mathematical reasoning.
d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non- overlapping parts, applying this technique to solve real world problems.

Digression on Meaning-Method-Mastery

Another example: equivalent fractions, which are given meaning in third grade (3.NF.3.b), method in fourth grade (4.NF.1) and are used to add fractions in fifth grade (5.NF.1) providing an opportunity to exhibit mastery.

Note that the Smarter Balanced claims parallel this structure.
Claim 1 is about "Concepts and Procedures" – that is, **meaning** and **method**.
Claims 2, 3 and 4 are about problem solving, reasoning and modeling, three aspects of **mastery**.
(That means that method alone will account for only about 20% of the assessment.)

Back to our story

The "double 23" task has a track record of engaging the mathematical practices, providing an opportunity to discuss properties and their relationship to algorithms, provides a clear example of algebraic thinking grounded in number work, as well as connects to the story of base-two representation.

Coherence, along with focus, means prioritizing activities with strong connections to many threads, and engaging the Mathematical Practices in the context of major work.

Third Exercise

In what order might "base twelve kids" learn their multiplication facts, in order to support using properties of arithmetic to connect those facts?

Fourth Exercise

Find the first four digits (including the ones digit) of the base two-imal expansion of $\sqrt{2}$.

Use your work to place $\sqrt{2}$ on a number line marked with two-imals, and to provide the best inequalities you can involving $\sqrt{2}$ and fractions with denominator 8.

Base *b*-imals

Base *b*-imals constitute a culmination of work on the number system in Math 21X. (A "pinnacle" in Jason Zimba's terminology.)

They bring together the work on fractions, and the whole-number place value work (as reflected by decimals being in the fraction strand in grade 4 and the numbers and operations strand in grade 5 in the Common Core).

Exercise/ discussion

Analyze the use of activities around converting fractions to b-imals as pinnacle activities in preservice content preparation.
That is, name such activities and discuss their role in preservice learning and ties to the Common Core.

Happy to share!

We are keeping our activities (including modifications back-and-forth) in a shared Dropbox. We already have shared with Lane Community College colleagues.

We are writing a text to support the use of such activities, hoping to eventually publish in some (semi-)open model.

On the in-service side

We have been seeing the fruits of the Lane Ignite partnership between us, as content experts, and local districts. We often co-facilitate with classroom experts, following a model which has proven successful in the development of the Common Core, Illustrative Mathematics, Smarter Balanced, work out of the University of Michigan and University of Arizona, the states of California and Idaho, SCASS...

We have served in whatever ways the districts' have identified as needs.

This has been in part supported by a gift which allows Tricia to devote up to 0.5 FTE to these efforts. Would we (higher-ed + ODE + districts) want to engage OEIB to see if some combination of private and public funding could scale this up over the whole state? Preliminarily, OEIB is receptive. TOTOM is a key community from which to build.